INTRODUCTION TO FOURIER ANALYSIS

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SUMMARY

Comprehensive, user friendly, and pedagogically structured

A fast, easy way to learn about the electrical engineer's most important mathematical tool

Based on a groundbreaking one-semester course originated by Professor Norman Morrison at the University of Cape Town, this book serves equally well as a course text and a self-study guide for professionals. Offering only relevant mathematics, it covers all the core principles of electrical engineering contained in Fourier analysis, including the time and frequency domains; the representation of waveforms in terms of complex exponentials and sinusoids; complex exponentials and sinusoids as the eigenfunctions of linear systems; convolution; impulse response and the frequency transfer function; magnitude and phase spectra; and modulation and demodulation.

- Covers Fourier analysis exclusively for electrical engineering students and professionals
- Offers a complete FFT system contained on the enclosed disks (long for IBM compatibles, the other for Macintosh)
- Includes dozens of examples drawn from electrical engineering
- Packed with exercises, samples, and end-of-chapter problem sets

CONTENTS

Preface xiii
Acknowledgments xviii
Nomenclature and Abbreviations xix

PART 1 CONTINUOUS FOURIER ANALYSIS

1 Background

1.1 Types of Problems
1.2 Historical Background

2 Fourier Series for Periodic Functions

2.1 Orthogonality of Vectors and Functions
2.2 The Complex Exponentials
2.3 Properties of the Complex Fourier Coefficients
2.4 Parseval's Theorem for Periodic Waveforms
2.5 Convergence of Fourier Series

Notes and Comments
Exercises

3 The Fourier Integral

3.1 Introduction
3.2 The Fourier Integral
3.3 What Does the Fourier Integral Mean?
3.4 Two Basic Theorems
3.5 Parseval's Theorem for Pulses
3.6 Existence of the Fourier Integral
3.7 Asymptotic Bounds for $F(\omega)$
4 Fourier Transforms of Some Important Functions

4.1 Introduction
4.2 The Rectangular Pulse
4.3 The Single-Sided Decaying Exponential
4.4 The Double-Sided Decaying Exponential
4.5 The Signum Function
4.6 The Dirac Delta or Unit Impulse
4.7 The Unit Step
4.8 The Eternal Complex Exponential
4.9 The Eternal Cosine and Sine Functions
4.10 Periodic Functions
4.11 The Periodic Impulse Train

Exercises

5 The Method of Successive Differentiation

5.1 The Differentiation Property
5.2 Differentiating Functions with Discontinuities
5.3 The Method of Successive Differentiation
5.4 One Small Complication and How to Resolve It
5.5 Non-polynomials Sections

Exercises

6 Frequency-Domain Analysis

6.1 Introduction
6.2 Response of a Linear Time-Invariant System to a Pulse Function
6.3 AC Circuit Analysis Using the Fourier Transform
6.4 Response of a Network to a Periodic Function
6.5 Finding the Transfer Function for Pulses

Exercises

7 Time-Domain Analysis

7.1 Introduction
7.2 The Impulse Response
7.3 Convolution
7.4 What Does the Convolution Product Mean?
7.5 Convolution the Graphical Way
7.6 Evaluating the Convolution Integral Analytically
7.7 Convolution in the Frequency Domain

Exercises

8 The Properties

8.1 The Linearity Property
8.2 The Realness Property
8.3 The Symmetry Properties
8.4 The Area Property
8.5 The Duality Property
8.6 The Reciprocal-scaling Property
8.7 The Time-shift Property
8.8 The Frequency-shift Property
8.9 Time-domain Differentiation
8.10 Frequency-domain Differentiation
8.11 Time-domain Convolution
8.12 Frequency-domain Convolution
8.13 Two Properties of the Dirac Delta

8.14 The Integration Property

Exercises

9 The Sampling Theorems

9.1 Introduction
9.2 Time-domain Impulse Sampling
9.3 Time-domain Analysis of the Recovery Process
9.4 Sampling with Pulses Other than Dirac Deltas
9.5 Sampling in the Frequency Domain

Exercises
10 The Discrete Fourier Transform 323

10.1 Introduction 323
10.2 The Discrete Complex Exponentials 323
10.3 The Discrete Fourier Transform 330
10.4 Properties of the DFT 338
Notes and Comments 341
Exercises 343

11 Inside The Fast Fourier Transform 348

11.1 Introduction 348
11.2 The FFT for Small Values of N 348
11.3 The General Radix-2 FFT Algorithm 354
11.4 Setting Up the Rules 358
11.5 The Complete FFT Flowchart 363
11.6 The Cooley-Tukey Derivation of the Radix-2 Algorithm 365
11.7 Final Comments 368

12 The Discrete Fourier Transform as an Estimator 372

12.1 Introduction 372
12.2 Relationships Based on the Rectangular Rule 373
12.3 Aliasing 378
12.4 The FFT as an Estimator for the CFTs 383
12.5 Inverting CFT Spectra Using the FFT 390
Exercises 395

13 The Errors in Fast Fourier Transform Estimation 399

13.1 Introduction 399
13.2 The Errors in the Estimates 400
13.3 Canonical Pulses and Order of Continuity 401
13.4 The Error Expressions 403
13.5 Some Properties of $E_N(n)$ 404
13.6 The Log-Linear Z-curves 406
13.7 Asymptotic Behavior of Noncanonical Functions 409
13.8 Break Points not at a Sampling Instant 415
13.9 Error Correction of FFT Estimates 418
13.10 Confirmation of Theoretical Results 419
13.11 Zero Padding Increases the Estimation Errors 419
Exercises 420

14 The Four Kinds of Convolution 432

14.1 Introduction 432
14.2 The Four Kinds of Convolution 435
14.3 The Span Restriction 442
14.4 Discrete Circular Convolution on the FFT 447
14.5 Discrete Linear Convolution Using the FFT 451
14.6 Continuous Linear Convolution Using the FFT 451
14.7 Continuous Circular Convolution Using the FFT 453
14.8 Operation Count to Perform Convolution 454
Exercises 457

15 Emulating Dirac Deltas and Differentiation on the Fast Fourier Transform 463

15.1 Time-domain Dirac Deltas on the FFT 463
15.2 Frequency-domain Dirac Deltas on the FFT 466
15.3 Differentiation on the FFT 467
Exercises 474

PART 3 THE USER'S MANUAL FOR THE ACCOMPANYING DISKS

Chapters 16 and 17 (Located in README files on the disks)

(A) Macintosh Disk 481
(B) DOS Disk 482

Appendix 1 References and Further Reading 485
Appendix 2

A. Three Short Tables of Fourier Transforms
B. The Properties

Answers to the Exercises

Index

TOP
Introduction. These notes are, at least indirectly, about the human eye and the human ear, and about a philosophy of physical phenomena. (Now don’t go asking for your money back yet! This really will be a mathematics - not an anatomy or philosophy - text.Â Mathematically, Fourier analysis has spawned some of the most fundamental developments in our understanding of infinite series and function approximation - developments which are, unfortunately, much beyond the scope of these notes. Equally important, Fourier analysis is the tool with which many of the everyday phenomena - the perceived differences in sound between violins and drums, sonic booms, and the mixing of colors - can be better understood.