Meeting times: Wednesday 9:30-1200, starting Sept 28.
Meeting place: Classics 11.

I will be away October 12 and 19: We may arrange another lecture, whose
title (subject-matter) will be announced (decided upon) later on.

There will be one lecture a week, with discussion. Following is a list of the
lectures, with required readings and, in curly brackets, suggested readings.
(Some of the latter may be too technical or in the wrong language for some
members of the class.) All of the required readings (at least) will be found on
e-reserve. Those taking the course for credit will be expected to produce a
paper relating to the subject matter of the course. I will read and comment
on drafts turned in by the end of eighth week, so that there will be an
opportunity to revise.

Set theory in its lower reaches, however disguised, is part of the warp and
woof of contemporary mathematics. But these lower reaches point irresistibly
(at least for those unwilling to close their eyes) to something more—to higher
infinities—that, however much of it one brings within the bounds of ordinary
mathematics by means of axioms, always spills over the bounds and demands
new axioms. Here is the essential incompleteness of our understanding that
was suggested by Kant with his antinomies, if not quite where he located it.

On what grounds do we accept new axioms? The question invites some
historical perspective on the mathematics of the infinite that we already
have, as well as philosophical perspective on the question of what constitutes “grounds”. And when we add to this question the reactionary challenges that essential incompleteness and the whiff of paradox associated with it engender, and the tendency to blend these with the historical resistance to the actual infinite, a need for both historical and conceptual clarity becomes even more apparent. The need was exacerbated by the wide misunderstanding, especially in the first half of the twentieth century (but spilling over into the present), of the essential incompleteness of set theory, leading to the so-called paradoxes of set theory.

This course will thus be a blend of philosophy, history (of the philosophy-through-history species) and a bit of mathematics. The main issues that we will discuss will not require much knowledge of the latter, but it will require a minimum skill in understanding elementary arguments. I blush somewhat at the broad strokes with which I will paint the history up to the nineteenth century in these lectures; but clear traces of influence go back at least to Greece in the fourth century B.C.: We have lots to cover.

Although we shall in time have discussed the axioms of set theory, this is not a course in axiomatic set theory. I list, in order of increasing demand on the reader, a few texts or treatises on that subject: [Enderton, 1977] (quite elementary) [Kunen, 1980] (axiomatic set theory and independence proofs: beautifully written) [Jech, 1978] (a treatise) [Kanamori, 1994](A very lively and attractive treatment, focusing on the investigation of axioms asserting the existence of large transfinite numbers).

**Lecture 1.** Introduction: Now and Then. (Sept. 28)

**Lecture 2.** Exact Science in Ancient Greece: Uncovering the Infinite. (Oct. 5)

**Readings:** Aristotle’s *Physics*, Book III, Ch. 4-8 and Book VI, Euclid’s *Elements*, Books I, V, X and XII. (Look at the definitions, postulates, common notions, and theorems at least), {[C.H. Edwards, 1979, pp. 10-19].}

**Lecture 3.** Sets, the Infinite, and Paradoxes in Late Medieval and Early Modern Times: Philosophy and Mathematics. Oct 26

**Readings:** [Murdoch, 1982], {[Duhem, 1985, Ch. 1-2]}, [Mancosu, 1996, Ch. 3-4], [Grattan-Guinness, 1980, Ch 2], An excerpt from [Berkely, 1834], {[C.H. Edwards, 1979, Ch. 8 and 9]. Primary sources concerning the origin and development of the calculus can be found in [Struik, 1969, Ch. 4-5].}
Lecture 4. Sets, Functions, and the Actual Infinite in Nineteenth Century Mathematics. (Nov. 2)
   Readings: [Grattan-Guinness, 1980, Ch 3 and (optional) 4], an excerpt from [Bolzano, 1851], [Ferreirós, 1999, Ch 1].

Lecture 5. The Foundations of Arithmetic and Analysis. (Nov 9) Readings: [Dedekind, 1887; Dedekind, 1872], {[Frege, 1884]}.

   Readings: [Cantor, 1874], [Cantor, 1883a], [Zermelo, 1908], {[Cantor, 1891]}.

Lecture 7. The so-called ‘Paradoxes of Set Theory’ and Reactions to It. (Nov. 23)
   Readings: An excerpt from [Hilbert, 1900], [Russell, 19308], [Baire, Borel, Hadamard and Lebesgue, 1905], [Weyl, 1921], [Bishop and Bridges, 1985, Ch. 1].

Lecture 8. Cumulative Hierarchies of Sets, Zermelo-Fraenkel Set Theory. (Nov. 30)
   Readings: [Zermelo, 1930], [Gödel, 1964], [Scott, 1974].

Lecture 9. Essential Incompleteness [the Absolute Infinite] and the Search for New Axioms. (Whenever)
   Readings: [Jensen, 1995], [Feferman, Friedman, Maddy and Steel, 2000, Especially the piece by J. Steel].

References

la théorie des ensembles, Bulletin de la Société Mathématique de France

Selected Readings, second edn, Cambridge University Press. First edition
1964.

Berkely, G. [1834]. The analyst, or, A discourse addressed to the infidel
mathematician, London: Jacob Thonson.


Set theory is a branch of mathematical logic that studies sets, which informally are collections of objects. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics. The language of set theory can be used to define nearly all mathematical objects. The modern study of set theory was initiated by Georg Cantor and Richard Dedekind in the 1870s. After the discovery of paradoxes in naive set theory, such as Russell's paradox, philosophy refers to broad and general questions: specialist articles which are now classed as philosophical logic will not be published. The Editor will consider articles on the relationship between logic and other branches of knowledge, but the component of logic must be substantial. Topics with no temporal specification are to be interpreted both historically and philosophically. The set of journals have been ranked according to their SJR and divided into four equal groups, four quartiles. Q1 (green) comprises the quarter of the journals with the highest values, Q2 (yellow) the second highest values, Q3 (orange) the third highest values and Q4 (red) the lowest values. Category. History. 2005. Q2. History.